Hydrogen Peroxide-IV. Reaction of Oxygen and Hydrogen Peroxide at a Gold Electrode," *Élektrokhimiya*, 6, 1522 (1970).

Tarasevich, M. T., and V. A. Boydanovskaya, "Influence of the State of Platinum Electrode Surface on Electroreduction of Molecular Oxygen-I. Oxidation of Electrode by Anodic Polarization," *ibid.*, 7, 1072 (1971).

Tikhomirova, V. I., V. I. Luk'yanycheva, and V. S. Bagotskii, "Use of a Rotating Platinum Disk Electrode with Preliminary Vacuum Pretreatment of the Surface for Study of the Ionization of Molecular Oxygen in an Alkali," *ibid.*, 3, 762 (1967).

Truesdale, G. A., A. L. Downing, and G. F. Lowden, "The Solubility of Oxygen in Pure Water and Seawater," *J. Appl. Chem.*, 5, 53 (1955).

Yuzhanina, A. V., V. I. Luk'yanycheva, N. A. Shumilova, and V. S. Bagotskii, "Investigation of the Mechanism of the Cathodic Reduction of Oxygen on Smooth Anodically-Cathodically Treated Platinum in Alkaline Solution," *Élektrokhimiya*, 6, 1054 (1970).

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Flow Maldistribution in Packed Beds: A Comparison of Measurements with Predictions

Experimental measurements are reported on flow maldistribution in packed beds containing side streams and deliberately created spatially nonuniform resistance to flow. The actual experimental technique involved the determination of the velocity field of the gas stream exiting the column through the use of a hot wire anemometer. The experimental measurements were compared with predictions based on the numerical solution of the differential vectorial form of the Ergun equation and reasonable agreement was obtained. Both the measurements and the analysis confirmed the existence of preferential flow in the vicinity of the walls even for uniformly packed beds.

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SCOPE

The quantitative understanding of flow nonuniformities in packed beds is of considerable practical importance in chemical reaction engineering. Flow maldistribution may occur due to spatially variable resistance to flow, as brought about by variable porosity or particle diameter; nonuniform flows will also occur when the fluid passing through the system is introduced in a nonuniform manner, for example, in the form of a side stream. In recent years some attention has been paid to these problems (Stanek and Szekely, 1972; Radestock and Jeschar, 1970), but this earlier work was almost exclusively analytical.

The work to be described in this paper was undertaken with a view of providing an experimental test of the vectorial differential form of the Ergun equation as applied to nonuniform flows in packed bed systems. In this paper extensive measurements of the velocity profiles in a gas stream exiting the top of a packed bed, containing spatially distributed, nonuniform resistance to flow are reported. The experimental measurements are then compared with theoretical predictions, based on an adaptation of the previously published vectorial, differential form of the Ergun equation. The modifications of this earlier theoretical treatment include the use of more realistic boundary conditions and an allowance for increased porosity in the vicinity of the walls and for the existence of possible side streams. The experimental verification of the vectorial, differential form of the Ergun equation is thought to be the major contribution of the paper; it follows that the Ergun equation should provide a sound basis for further modeling studies aimed at more complex situations where flow maldistribution is accompanied by heat and mass transfer, for example, hot spot formation.

Fluid flow through packed beds which have a spatially variable resistance to flow is of considerable practical importance in chemical and metallurgical reaction engineering. Spatial nonuniformity of resistance to flow will occur to some extent in all packed-bed systems because the region in the immediate vicinity of the wall has a higher porosity than the bulk, which is known to result in preferential flow. Local variations in the resistance to flow may also be caused by nonuniform packing (that is, variable void fraction) or by the segregation of particles of different size during filling. Finally, non-

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uniform, or nonparallel flow will also occur, even in homogeneous beds, if the fluid passing through the system is introduced in a nonuniform manner, for example, through side streams.

Because of the obvious importance of these phenomena, various aspects of this problem received considerable attention. The structure and void space distribution in randomly packed beds of solids was studied by several workers including Furnas (1929) and Benenati and Brosilow (1962). In a now classical paper, Furnas has shown that the void fraction of a packed bed, made up of solids of different sizes, may show a marked variation depending on the particle size ratio and on the relative proportion of the solid components. Furnas was one of the first investigators to note that for packed beds the void fraction was larger in the vicinity of the wall than in the bulk of the bed. Benenati and Brosilow, by performing elegant experiments, established a more precise relationship for the spatial dependence of the void fraction in the vicinity of the wall.

The directional permeability of packed beds has been measured by Rice et al. (1970). The actual experimental measurement of gas velocities for nonuniform flows in packed beds has been confined to studying the wall effect in uniformly packed beds in parallel flow. Numerous investigations have been conducted using hot wire anemometers to determine the velocity profile at the outlet from the bed, and there appears to be general agreement regarding the existence of preferential flow in the vicinity of the walls. This previously cited work was primarily experimental although Price (1967) and Standish (1973) did attempt an interpretation of their results in terms of expressions derived from an overall momentum balance.

An alternative approach to representing flow maldistribution in uniformly packed beds was provided by the use of radial dispersion, a good example for which is the recent work of Bischoff and Schertz (1969).

Regarding theoretical work on flow maldistribution, it has been recognized in recent articles that the proper representation of nonuniform flow in packed beds requires the use of the multidimensional Ergun equation in a differential form. In these papers, computed results have been presented for a range of nonuniform flows and geometries which were based on the differential vectorial form of the Ergun equation. However, up to the present, no direct experimental verification has been available of these computed results. The purpose of the work to be described in this paper is to perform the experimental measurement of nonuniform flows through packed beds for a range of packing configurations and flow geometries which could then be tested against theoretical predictions based on the previously presented analysis.

APPARATUS AND EXPERIMENTAL PROCEDURE

The apparatus was so constructed to allow the ready measurement of the velocity profile in a gas stream exiting a packed bed which could be packed with a variety of solid particles and into which gas could be fed from a number of inlet ports. A schematic diagram of the apparatus is shown in Figure 1. It is seen that compressed air could be supplied to the bed at a metered rate; before entering the bed through one of the inlet ports, the gas was passed through a flow straightener for the case of parallel flow and then through an empty chamber, where facilities were provided for measuring the pressure profile at the inlet to the column. The knowledge of this pressure profile was thought to be useful in establishing the boundary conditions for the modeling equations.

Two columns were used in the measurements; a larger unit, 15.25 cm in diameter, packed to a height of about 60 cm, which was used in the preliminary studies and a smaller column, 10.1 cm in diameter which was packed to a height of 30.5 cm. A total of 10 inlet taps were available on the larger column and some 4 inlet taps were provided on the smaller unit; details of the inlet tap configuration will be given subsequently.

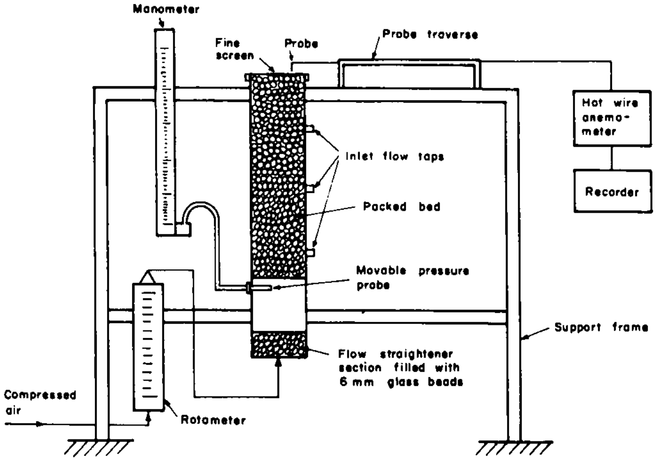

Fig. 1. Experimental apparatus for the measurement of nonuniform flow in packed beds.

The solid particles contained in the bed were supported on a 10 mesh wire screen, and a finer (200 mesh) screen was also employed at the top of the column to retain the solids in the bed.

The velocity profile of the gas stream leaving the bed was measured by using a calibrated DISA hot wire probe, mounted on a suitable traversing mechanism. The probe itself was located about 1 cm above the outlet of the bed and was connected to a DISA 55DO5 anemometer, the output of which was passed through a linearizer and then recorded on a Leeds and Northrup Strip Chart Recorder.

It is to be noted that in most previous investigations circumferential probes were used for the measurement of preferential flow in the vicinity of the walls. While this arrangement is certainly attractive for smoothing and averaging the velocity measurements, it could not be used in the present case because many of the systems studied produced asymmetric flow fields.

Experimental Procedure

For uniform packing, glass beads were poured into the column until the desired height was reached; then the air was turned on, which allowed the particles to settle and more solids were added to complete the procedure. The actual void fraction of the bed was ascertained independently by weighing the solids that were supplied to the system.

Nonuniform packing of the bed was accomplished by placing a vertical tube into the bed which was then filled with a given solid material. The remainder of the bed was filled with solids the size of which differed from those already contained in the bed and the tube was carefully withdrawn. In some of the runs the maldistributed beds were prepared by filling adjacent sections of the bed, separated by a vertical screen.

In an actual experimental run, after the solids were charged, the air was turned on, the flow rate was set at predetermined value and the outlet surface of the bed was traversed by the hot wire probe. The values of the pressure drop across the bed ranged from 0.005 to 0.059 atm and the linear gas velocities measured ranged from 17.1 to 61.6 cm/s. In addition to the measurement of the velocity profile, a pressure traverse across the inlet was also done for all the experimental runs.

Experimental Configurations

Glass spheres, 0.1, 0.3, and 0.6 cm in diameter were used as the packing material; the packing was so arranged that any given portion of the bed contained uniformly sized particles only.

In order to provide a severe test of the expressions developed for nonuniform flow, it was thought desirable to examine a broad range of packing configurations and gas inlet conditions. The actual packing arrangements and inlet nozzle configurations employed in the experimental program are illus-

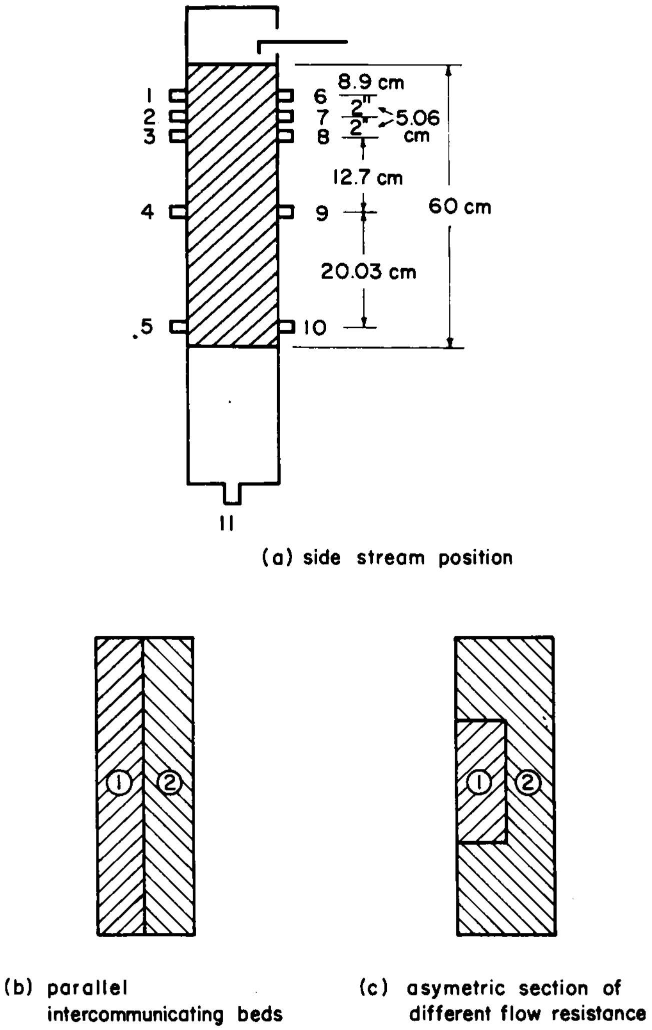

Fig. 2. Sketch of the packing and of the inlet nozzle arrangements—large column.

Legend: Region 1—packed with 0.3-cm diameter glass beads.
Region 2—packed with 0.6-cm diameter glass beads.

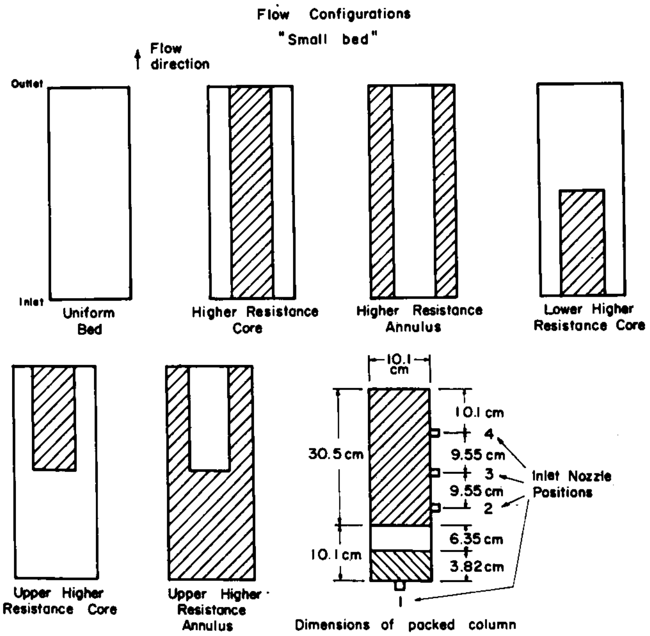

Fig. 3. Sketch of the packing and of the inlet nozzle arrangements—small column.

Legend: The hatched areas are packed with 0.1-cm diameter glass beads while the blank areas are packed with 0.3-cm diameter glass beads.

trated in Figures 2 and 3.

It is noted that for the system depicted in Figure 2 (the 15-cm diameter column) regions 1 contained glass spheres 0.3 cm in diameter, whereas regions 2 contained glass spheres 0.6 cm in diameter; thus regions 1 represented the zones with the higher resistance to flow.

It is seen in Figure 3 that a broad range of packing configurations were employed in the experiments conducted with the 10-cm diameter column; here the hatched zones denote regions which were packed with particles 0.1 cm in diameter, while the blank zones correspond to regions packed with particles 0.3 cm in diameter. It follows that the hatched zones designate the regions which offered a higher resistance to flow.

Some typical, experimentally measured velocity profiles are shown in Figures 4 and 5. Further measurements will be presented in the subsequent section and additional data are available in the thesis by Poveromo (1975).

Figure 4 shows experimentally measured velocity profiles for a column packed with two types of spheres, as indicated on the inset, for two distinct mass flow rates.

The following observations may be made:

1. The velocity profiles are similar for the two different mass flow rates, which is consistent with earlier discussions based on the theoretical work of Stanek and Szekely (1974), provided the inertial component dominates the resistance flow.

2. The velocity profile exhibits a pronounced minimum in the central vertical plane separating the two different types of packing. This behavior may be explained by considering the fact that the local void fraction is likely to show a minimum at the interface separating particles of different size. Such a finding has been reported by Furnas for the bulk void fraction of mixed solids.

3. Inspection of Figure 4 also shows quite marked local maxima in the velocity profile in the vicinity of the wall. This finding is consistent with that of earlier investigators and is attributable to the local maxima in void fraction in the wall region. Of course, the velocity has to be zero at the surface of the wall itself.

4. Finally, it is also seen that the average velocity appears to be larger on the right-hand side of the figure, which is consistent with the fact that the overall resistance to flow as lower in this region.

Figure 5 shows a plot of the normalized (with respect to the average) velocity profile for a packing where the bed containing a core with a higher resistance to flow, for four distinct gas inlet arrangements, as sketched on the inset.

It is readily seen that for the system considered the location of the inlet nozzle had no effect on the velocity profile at the outlet and one may infer from these measurements that parallel flow had been achieved by the time the gas reached the outlet for these particular conditions.

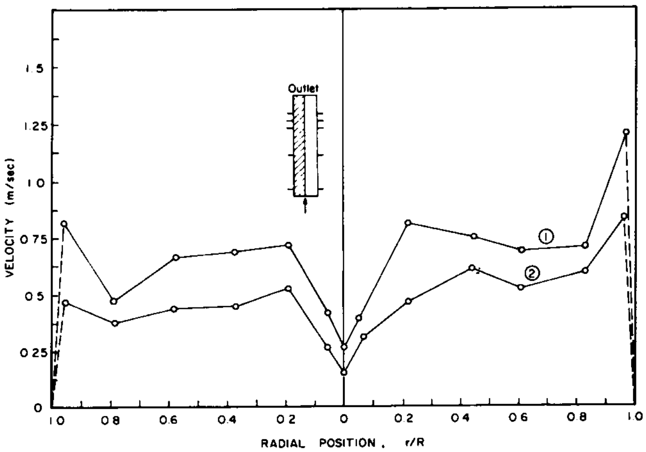

Fig. 4. The velocity profile for flow through parallel intercommunicating beds, parallel flow from inlet nozzle 11, large column

Curve	Flow rate	Pressure drop, atm
1	$8.42 \times 10^{-3} m^3/s$	0.058
2	$7.02 \times 10^{-3} m^3/s$	0.034

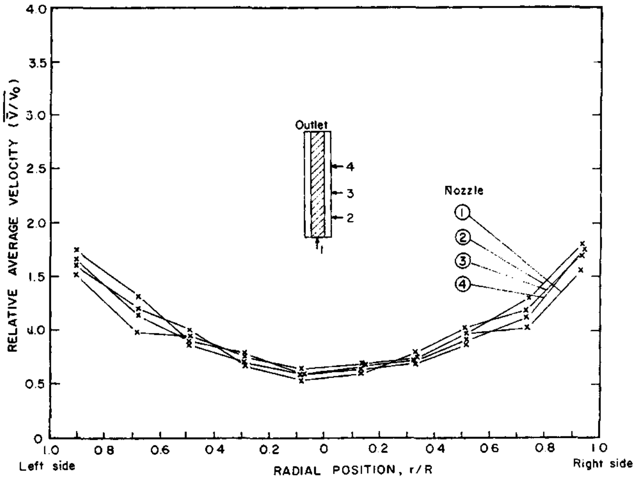

Fig. 5. The velocity profile for flow through high resistance core configuration, where the core is packed with 0.1-cm diameter glass beads and the annular region with 0.3-cm diameter glass beads. The numbers on the curves indicate the inlet nozzle positions.

Inspection of Figure 5 also shows the wall effect which is accentuated in the present case by having the region of lower resistance forming an annular space in the vicinity of the walls.

COMPARISON OF THE MEASUREMENTS WITH PREDICTIONS BASED ON THE VECTORIAL FORM OF THE ERGUN EQUATION

In this section we shall present a direct comparison between the experimentally measured velocity profiles with those predicted on the basis of the vectorial form of the Ergun equation which was described in an earlier paper by Stanek and Szekely (1974). However, before we proceed with this comparison it is worthwhile to present a brief recapitulation of the principal governing equations describing two-dimensional fluid flow through packed beds.

Fluid Flow Equations

In its vectorial form the Ergun equation may be written as

$$-\nabla P = \mathbf{V}\,(f_1 + f_2 V) \tag{1}$$

It has been shown that when $N_{Re} > 150$, the Ergun equation is dominated by the inertial term and under these conditions f_1 may be neglected. Let us assume, furthermore, incompressible flow and then, in order to eliminate the pressure term from Equation (1), let us operate with the $\nabla \times$ operator on both sides of Equation (1). Thus we obtain

$$\nabla \times \mathbf{V} - \mathbf{V} \times \nabla\,[\ln(f_2 V)] = 0 \tag{2}$$

The components of the velocity vector also have to satisfy the equation of continuity, thus,

$$\nabla \bullet \mathbf{V} = 0 \tag{3}$$

Upon finding the velocity field through the solution of Equations (2) and (3) for the appropriate boundary conditions, we can then evaluate the pressure distribution from

$$\nabla \bullet \nabla P = -\,\mathbf{V} \bullet \nabla (f_2 V) \tag{4}$$

For the purpose of computation, it is more convenient to work in terms of the stream function which for cartesian coordinates is defined as

$$V_x = \frac{\partial \psi}{\partial y} \qquad V_y = -\frac{\partial \psi}{\partial x} \tag{5)-(6}$$

where y designates the coordinate parallel to the axis of the column and x designates the direction perpendicular to y.

Thus the two-dimensional form of Equation (2) may be written as follows:*

$$\begin{aligned}
&\frac{\partial^2\psi}{\partial x^2}\left[2\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2\right] \\
&\quad + \frac{\partial^2\psi}{\partial y^2}\left[2\left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2\right] \\
&\quad + \left[\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2\right]\left[\frac{\partial\psi}{\partial y}\,\frac{\partial(\ln f_2)}{\partial y}\right. \\
&\quad \left. + \frac{\partial\psi}{\partial x}\,\frac{\partial(\ln f_2)}{\partial x}\right] + 2\frac{\partial^2\psi}{\partial x\partial y}\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial\psi}{\partial y}\right) = 0
\end{aligned} \tag{7}$$

It is to be noted furthermore that Equation (7) does remain unchanged, formally, if rendered dimensionless by using a mean velocity V_0 and a column diameter D.

Regarding the boundary conditions, these had to express the fact that the side walls were impervious to flow (except when side stream nozzles were used) which could be expressed by setting:

$$\psi = \text{constant} \quad \text{at} \quad X = 0,\ X = 1 \tag{8)-(9}$$

where X now denotes the dimensionless spatial coordinate.

The boundary conditions at the inlet and at the exit of the column were established with the aid of the experimental measurements which indicated that the pressure was uniform both at the inlet and at the exit. This finding was expressed by stating

$$\frac{\partial \psi}{\partial y} = 0 \quad \text{at} \quad y = 0,\ y = L/D \tag{10)-(11}$$

where L is the length of the column. It is to be noted that Equations (10)-(11) have to be regarded as an approximation adopted for the sake of mathematical convenience because, strictly speaking, the Ergun equation will not be valid at the boundaries.

In performing the actual computation, the spatial nonuniformities in the resistance to flow were expressed by assigning an appropriate position dependence to f_2. The following particular cases were considered:

1. In accounting for the increased porosity near the walls, the experimental measurements of Benenati and Brosilow (1962) were used to estimate the void fraction at the grid point adjacent to the wall.
2. Appropriate allowance was made for the different bulk resistances of regions composed of different particles.
3. An approximate allowance was also made for the sharp decrease in the local porosity at the interface of two regions composed by the particles of different size—through the use of Furnas' (1929) correlation. The principal experimental variables are summarized in Table 1.

Equation (7) was put in a finite difference form and solved by the use of relaxation techniques using the CDC 6400 digital computer of the State University of New York at Buffalo. An 11×40 grid was used and a typical run required about 100 seconds of computer time. Further computational details, including a complete program listing, are available in the thesis upon which this paper is based (Poveromo, 1975).

* The equivalent expression to Equation (7) in cylindrical coordinates has been given in Stanek and Szekely (1974).

Table 1. The Principal Experimental Variables

Particle shape	Spherical
Void fraction in the bulk	0.38
Range of particle diameter investigated	0.1 cm-0.6 cm
Range of particle Reynolds numbers studied	100-400

Porosity Data

In the computation an allowance was made for the larger porosity in the vicinity of the walls (Benenati and Brosilow, 1962) by assigning the following values to the void fraction at the grid point adjacent to the walls, as a function of the D/d_p ratio:

D/d_p	ϵ_{wall}
25.4*	0.49
50*	0.47
36**	0.48
108**	0.45

The local minima in the void fraction at the interface of regions composed of particles of different diameters was estimated on the basis of Furnas' (1929) observations, as follows:

Ratio of particle diameters	$\epsilon_{interface}$
0.5	0.365
0.33	0.327

* Column diameter = 15.25 cm.
** Column diameter: 10.1 cm.

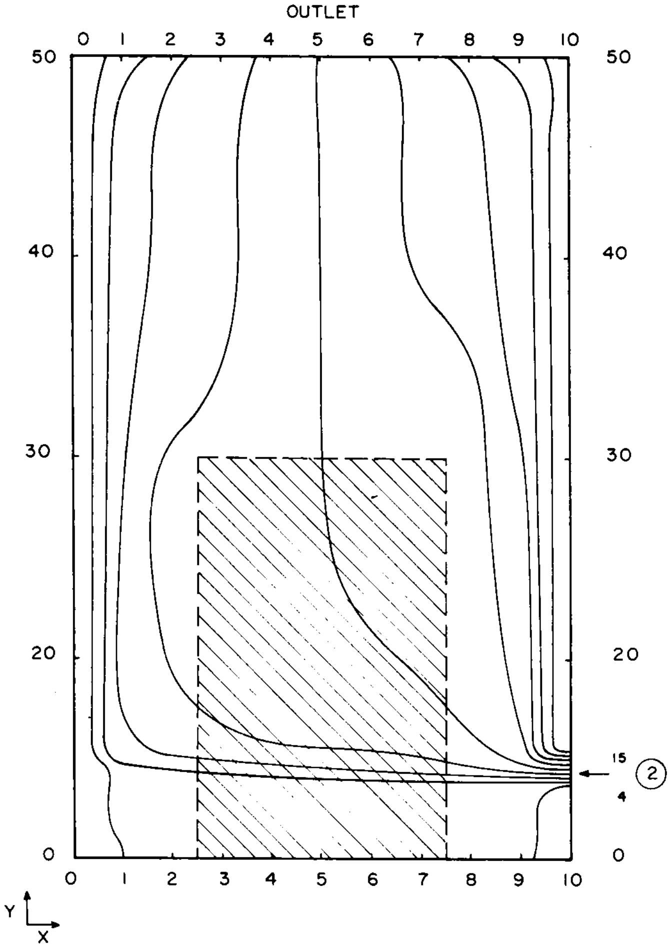

Fig. 6. Computed streamlines for flow from side nozzle 2 through a bed packed with a core of higher resistance (0.1-cm diameter glass beads) in the lower half of the bed. The rest of the bed is packed with 0.3-cm diameter glass beads.

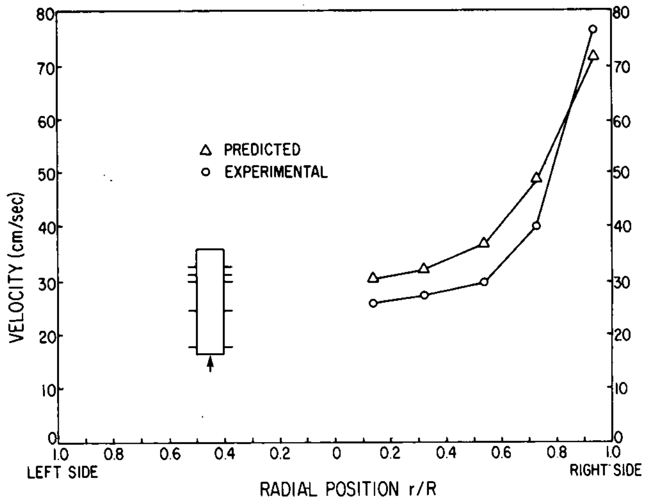

Fig. 7. Comparison of predicted and experimental outlet velocity profiles for parallel flow through a bed packed with 0.6-cm diameter glass beads. Inlet flow from nozzle 11, large column, flow rate, $8.42 \times 10^{-3} m^3/s$, pressure drop 0.047 atm.

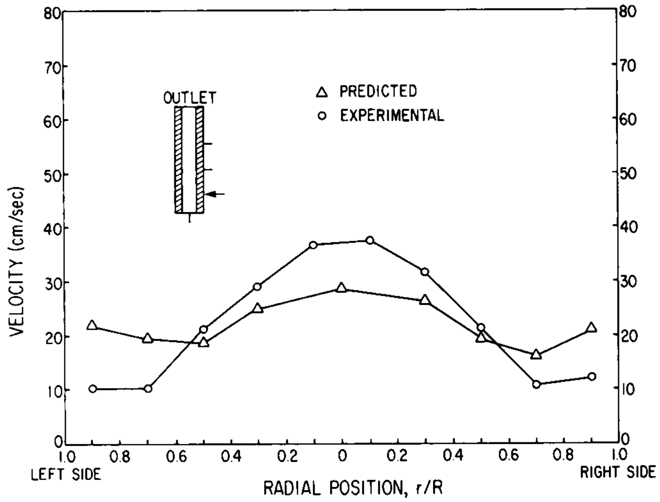

Fig. 8. Comparison of predicted and experimental outlet velocity profiles for side stream flow (nozzle 2, small column) through a bed packed with a higher resistance annulus (0.1-cm glass beads). The core is packed with 0.3-cm diameter glass beads. Flow rate, $1.77 \times 10^{-3} m^3/s$ pressure drop, 0.031 atm.

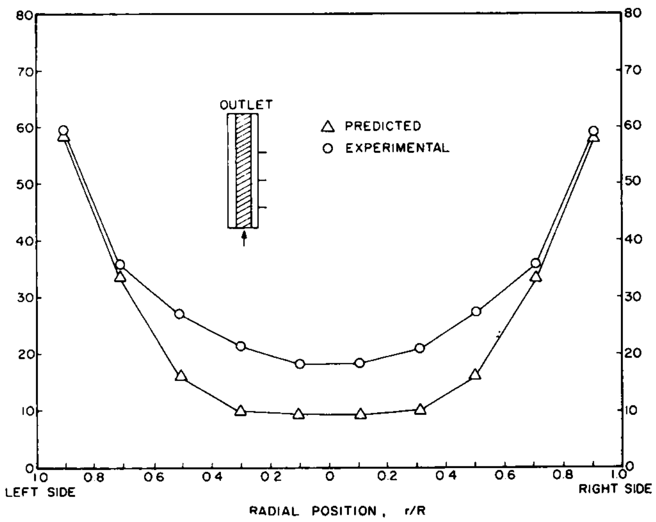

Fig. 9. Comparison of predicted and experimental outlet velocity profiles for parallel flow through a bed packed with a higher resistance core (0.1-cm diameter glass beads). The rest of the bed is packed with 0.3-cm diameter glass beads. Inlet flow from nozzle 1, small column, flow rate, $3.51 \times 10^{-3} m^3/s$, pressure drop, 0.035 atm.

Figure 6 shows the computed streamlines for a complex flow field, for a system with side stream injection and where the shaded area denotes a core with a higher resistance to flow. The pronounced wall effect, with the closely bunched streamlines indicating preferential flow, is apparent, together with the fact that parallel flow appears to have been approached at the exit from the system.

Comparison of Measurements and Predictions

Before proceeding with the presentation of the comparison between the measurements and the computed results, it should be noted that the cylindrical form of the equations was used for the systems depicting cylindrical symmetry, while a two-dimensional, cartesian system was used for representing the asymmetric situations. The computation of a truly three-dimensional flow field would have required an unduly large amount of computer time. It is thought, moreover, that the results would be representative of the behavior of a two-dimensional slice cut through the center of the column parallel to its axis.

A selection of the experimental measurements is shown in Figures 7 to 9; additional data are available in the thesis upon which this work is based. Figure 7 shows the behavior of a 15 cm wide column, packed uniformly with 0.6-cm glass spheres in parallel flow. There is reasonable agreement between measurements and prediction; the wall effect is also quite apparent.

The system depicted in Figure 8 involve the use of a side stream nozzle and a bed where the central core had a lower resistance to flow. The agreement between measurements and predictions is again quite reasonable; it is seen, furthermore, that the flow appears to be parallel at the outlet.

Finally, Figure 9 depicts a situation where the central core of the bed has a higher resistance to flow than the outer shell. It is seen that both predictions and measurements are in reasonable agreement and preferential flow is accentuated under these conditions. It is noted that while there is a discrepancy between the numerical values of the velocity in the central portion of the bed the actual overall volumetric flow rate is reasonably well predicted because the agreement is quite good over the major portion of the area available for flow.

DISCUSSION

Experimental measurements were reported on nonuniform flow through packed beds, as caused by wall effects, nonuniform packing, and the introduction of side streams. It was found that the measurements were in reasonable agreement with predictions obtained from the numerical solution of the two-dimensional, vectorial form of the Ergun equation. If follows that the vectorial form of the Ergun equation would provide a sound basis for representing flow maldistribution in packed bed reactors and consequently hot spot formation. It is noted that quite a marked preferential flow was found in the vicinity of the walls, and it is thought that this type of flow maldistribution is likely to occur in most packed-bed reactors.

Experimental measurements were also made of the pressure distribution at the inlet and at the exit of the column. It was found that for the experimental conditions the pressure was uniform, a fact which provided direct experimental proof for the validity of the boundary conditions used at the exit and at the inlet.

In closing it would be worthwhile to comment on the limitations of the work. The most important limitation is due to the fact that the Ergun equation (just as the Darcy Equation) is a macroscopic relationship. It follows that the use of this equation, or the use of relationships derived from it, becomes questionable when applied to distances that are comparable to the size of the particles that make up the packing. Another important limitation of the Ergun equation, as cited by Gauvin and Katta (1973), is the fact that it is not appropriate for systems containing particles of low sphericity.

The finding regarding the uniformity of the pressure at the inlet and at the exit is necessarily restricted to the range of experimental conditions employed in this study. The use of a two-dimensional, cartesian coordinate system to represent flow maldistribution in an essentially three-dimensional slice, is also an approximation. It is thought, however, that the measurements reported in the study do provide firm support for the use of the vectorial form of the Ergun equation for representing flow maldistribution phenomena in packed bed reactor systems. Further work would seem to be desirable in two areas. A better understanding of variations in local porosity at the interface separating regions composed of different packing materials would be helpful because such effects could contribute to hot spot formation. Finally, it would be desirable to apply the results of the flow study reported here to real reactor systems.

CONCLUSIONS AND SIGNIFICANCE

Experimental measurements on flow maldistribution in packed beds containing side streams and a deliberately created spatially nonuniform resistance to flow were reported. The principal findings of the study may be summarized as follows:

Preferential flow was found in the vicinity of the walls even for uniformly packed beds.

A sharp minimum in the local gas velocity was found in the vicinity of any vertical surface separating regions in the bed containing particles of different sizes.

For the experimental conditions, the normalized (relative) velocity profiles at the exit were qualitatively similar for each flow rate which is consistent with the fact that the inertial component of the Ergun equation provided the principal resistance to flow.

For the particular conditions examined, a uniformly packed bed with a height-to-diameter ratio larger than one was found to act as a flow straightener which evened out any nonuniformities introduced upstream. The experimental measurements were found to be in reasonable agreement with predictions based on the differential vectorial form of the Ergun equation for a broad range of packing geometries and inlet conditions.

The principal significance of the work described here is that it provides direct experimental verification for the differential, vectorial form of the Ergun equation which should provide a sound starting point for representing more complex problems where flow maldistribution may be accompanied by heat or mass transfer processes.

ACKNOWLEDGMENT

The authors wish to thank the American Iron and Steel Institute for partial support of this investigation under Grant 31-239. Thanks are also due to Mr. Mark Propster of the Center for Process Metallurgy for help with the computation.

NOTATION

d = particle diameter
D = column diameter
f_1 = $150\mu(1-\epsilon)^2/d^2\epsilon^3$, resistance parameter
f_2 = $1.75\rho(1-\epsilon)/d\epsilon^3$, resistance parameter
L = bed depth
N_{Re} = Reynolds number
R = column radius

$\mathbf{V}$, V, V_x, V_y = velocity vector, absolute magnitude value and component value, respectively
V_0 = average superficial velocity
x, y = rectangular coordinates
$X = x/D$ = dimensionless coordinate
$Y = y/D$ = dimensionless coordinate

Greek Letters

ϵ = porosity (void fraction)
∇ = gradient
μ = dynamic viscosity
$\psi,(\psi^* = \psi/V_0D)$ = dimensionless and dimensionless stream function, respectively
ρ = density

Subscripts

0 = average quantity

LITERATURE CITED

Benenati, R. F., and C. B. Brosilow, "Void Fraction Distribution in Beds of Spheres," *AIChE J.*, **8**, 359 (1962).

Furnas, C. C., "Flow of Gases Through Beds of Broken Solids," Bull 307, U.S. Bureau of Mines (1929).

Gauvin, W. H., and S. Katta, "Momentum Transfer Through Packed Beds of Various Particles in the Turbulent Flow Regime," *AIChE J.*, **19**, 775 (1973).

Poveromo, J. J., "Raceway and Channeling Phenomena in the Iron Blast Furnace," Ph.D. thesis, St. Univ. of New York, Buffalo (1975).

Price, J., "A Review of Fluid Flow Distribution Through Randomly Packed Beds," Aus. Atomic Ener. Comm. Rep., AECC/E178 (1967).

———., "The Distribution of Fluid Velocities for Randomly Packed Beds of Spheres," *Aus. Inst. Engrs. Mech. Chem. Eng. Trans.*, No. 1, 7 (1968).

Radestock, J., "Theroretische Untersuchung der Stationaeren Inkompressiblen und Kompressiblen Stroemung in Ruhenden, Geschichteten und Isotropen Schuettunge," Dissertation, Fak. Bergbau, Huettenwesen und Maschinenwesen der Techn. Univ. Clausthal (1969).

———., and R. Jeschar, "Ueber die Stroemung durch die Hochofenschuettung," *Stahl Eisen*, **22**, 1249 (1970).

———., "Theoretische Untersuchung der Inkompressiblen und Kompressiblen Stroemung durch Reaktor-Schuettungen," *Chem. Ing. Tech.*, **43**, 355 (1971a).

———., "Theoretische Untersuchung der Gegenseitigen Beeinflussung von Temperatur und Stroemungsfeldern in Schuettungen," *ibid.*, **43**, 1304 (1971b).

Rice, P. A., et al., "Anisotropic Permeability in Porous Media," *Ind. Eng. Chem.*, **62**, 23 (1970).

Schertz, W. W., and K. B. Bischoff, "Thermal and Material Transport in Nonisothermal Packed Beds," *AIChE J.*, **15**, 597 (1969).

Standish, N., and R. Newell, "Velocity Distribution in Rectangular Packed Beds and Non Ferrous Blast Furnaces," *Trans. AIME*, **4**, 1851 (1973).

Stanek, V., and J. Szekely, "The Effect of Non-uniform Porosity in Causing Flow Maldistributions in Isothermal Packed Beds," *Can. J. Chem. Eng.*, **50**, 9 (1972).

———., "Flow Maldistribution in Two-Dimensional Packed Beds Part II: The Behavior of Non-isothermal Systems," *ibid.*, **51**, 22 (1973).

———., "Three-Dimensional Flow of Fluids Through Nonuniform Packed Beds," *AIChE J.*, **20**, 974 (1974).

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Instability of Film Coating of Wires and Tubes

The stability problem of the free coating of wires and tubes by withdrawal is formulated and solved. The necessary condition for the stability of the film coating is given explicitly in terms of the Reynolds number, wave number, the Weber number, and implicitly in terms of the withdrawal velocity. The outcome of the competition between the destabilizing capillary pinching and the stabilizing capillary restoring force associated with the film thickness variation is shown to dictate the stability of the film. The comparisons between the present theoretical results and the known experimental results for falling films and creeping annular threads of viscous liquids are good. A possible application of the present results to some aspects of the qualitative design of a coating process is given.

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SCOPE

The film flows down a solid surface under the simultaneous actions of the gravity, the surface tension, and the viscous drag are frequently encountered in many important industrial processes. Such processes include: film coating of photographic paper or plates, cleaning, draining, coating of insulation on a wire, and the protection coating of tube walls, etc. A smooth and uniform film coating is usually difficult to attain because of the flow instability. The instability frequently either sets a limit on the production rate or dictates the selection of the material in precision coatings. A predictive theory of film instability is therefore of considerable practical significance. The purpose of this paper is to offer such a stability theory for the particular film flows encountered in the free coatings of circular wires or tubes.